Khalfin's Theorem and neutral mesons subsystem*

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February 2, 2008

Abstract

We analyze the proof of the Khalfin Theorem for neutral meson complex. The consequences of this Theorem are discussed: using this Theorem we find, eg., that diagonal matrix elements of the exact effective Hamiltonian for the neutral meson complex can not be equal if CPT symmetry holds and CP symmetry is violated. The Properties of time evolution governed by a time–independent effective Hamiltonian acting in the neutral mesons subspace of states are considered. By means of the Khalfin's Theorem we show that if such Hamiltonian is time–independent then the evolution operator for the total system containing neutral meson complex can not be a unitary operator. Within a given specific model we examine numerically the Khalfin's Theorem. We show for this model in a graphic form how the Khalfin's Theorem works. We also show for this model how the difference of the mentioned diagonal matrix elements of the effective Hamiltonian varies in time.

PACS numbers: 03.65.Ca, 11.30.Er, 11.10.St, 14.40.Aq
Keywords: *CP violation, CPT symmetry, neutral mesons.*

*Talk given at *13th Lomonosov Conference on Elementary Particle Physics*, Moscow, August 23 – 29, 2007.

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1 Introduction

One of the most interesting two state (or two particle) subsystems is the neutral mesons complex. The standard method used for the description of the properties of such complexes is the Lee–Oehme –Yang (LOY) approximation [1] – [7]. The source of this approximation applied by LOY to the description and analysis of the decay of neutral kaons is the well known Weisskopf–Wigner (WW) theory of the decay processes [8]. Within this approach the solutions of the Schödinger equation

$$i \, \frac{\partial |\psi ; t\rangle}{\partial t} = H \; |\psi ; t\rangle, \qquad |\psi ; t=0\rangle = |\psi_0\rangle, \tag{1}$$

(where H is the total selfadjoint Hamiltonian for the system containing neutral kaons and units $\hbar = c = 1$ are used) describe time evolution of vectors $|\psi ; t\rangle$ in the Hilbert space, $\mathcal{H}$, of states $|\psi ; t\rangle, |\psi_0\rangle \in \mathcal{H}$ of the total system under considerations and the Hamiltonian H for the problem is divided into two parts $H^{(0)}$ and $H^{(1)}$:

$$H \; = \; H^{(0)} + H^{(1)}, \tag{2}$$

such that $|K_0\rangle \equiv |\mathbf{1}\rangle$ and $|\overline{K}_0\rangle \equiv |\mathbf{2}\rangle$ are discrete eigenstates of $H^{(0)}$ for the 2–fold degenerate eigenvalue m_0,

$$\begin{aligned} H^{(0)}|\mathbf{j}\rangle &= m_0|\mathbf{j}\rangle, \;\; (j = 1, 2), \\ H^{(0)}|\varepsilon, J\rangle &= \varepsilon \, |\varepsilon, J\rangle, \end{aligned} \tag{3}$$

(where $\langle \mathbf{j}|\mathbf{k}\rangle = \delta_{jk}$ and $\langle \varepsilon', L|\varepsilon, N\rangle = \delta_{LN} \;\; \delta(\varepsilon - \varepsilon')$, $\langle \varepsilon, J|\mathbf{k}\rangle = 0$, $j, k = 1, 2$) and $H^{(1)}$ induces the transitions from these states to other (unbound) eigenstates $|\varepsilon, J\rangle$ of $H^{(0)}$ (here J denotes such quantum numbers as charge, spin, etc.), and, consequently, also between $|K_0\rangle$ and $|\overline{K}_0\rangle$. So, the problem which one usually considers is the time evolution of an initial state, which is a superposition of $|\mathbf{1}\rangle$ and $|\mathbf{2}\rangle$ states [1].

In the kaon rest–frame, this time evolution for $t \geq t_0 \equiv 0$ is governed by the Schrödinger equation (1), whose solutions $|\psi ; t\rangle$ have the following form [1, 4, 5]

$$|\psi ; t\rangle = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle + \sum_{J,\, \varepsilon} F_J(\varepsilon ; t)|\varepsilon, J\rangle, \tag{4}$$

where

$$|a_1(t)|^2 + |a_2(t)|^2 + \sum_{J,\, \varepsilon} |F_J(\varepsilon, t)|^2 = 1, \tag{5}$$

$$F_J(\varepsilon; t = 0) = 0. \tag{6}$$

Here $|F_J; t\rangle \equiv \sum_\varepsilon F_J(\varepsilon; t)|\varepsilon, J\rangle$ represents the decay products in the channel J.

Inserting (4) into the Schrödinger equation (1) leads to system of coupled equations for amplitudes $a_1(t)$, $a_2(t)$ and $F_J(\varepsilon; t)$. Adopting the WW approximations to these equations and solving them LOY obtained their approximate equations for $a_1(t), a_2(t)$ [1, 4, 9]. This gives, e.g. that [1],

$$i\frac{\partial a_1(t)}{\partial t} = h_{11}^{LOY}\, a_1(t) \; + \; h_{12}^{LOY}\, a_2(t), \tag{7}$$

where $t \gg t_0 = 0$, and

$$h_{jk}^{LOY} = m_0 \delta_{jk} - \Sigma_{jk}(m_0) \equiv M_{jk}^{LOY} - \frac{i}{2}\Gamma_{jk}^{LOY}, \;\; (j, k = 1, 2), \tag{8}$$

$$\Sigma_{jk}(x) = \sum_{J,\,\varepsilon} H_{jJ}^{(1)}(\varepsilon)\frac{1}{\varepsilon - x - i0}H_{Jk}^{(1)}(\varepsilon) = \langle \mathbf{j}|\Sigma(x)|\mathbf{k}\rangle, \;\; (j, k = 1, 2). \tag{9}$$

A similar equation can be obtained for $a_2(t)$.

Matrix elements h_{jk}^{LOY} form (2×2) matrix H_{LOY},

$$H_{LOY} \equiv M_{LOY} - \frac{i}{2}\,\Gamma_{LOY}, \tag{10}$$

(where $M_{LOY} = M_{LOY}^{+}$, $\Gamma_{LOY} = \Gamma_{LOY}^{+}$), acting in two–dimensional subspace (let us denote it by $\mathcal{H}_{||}$) of $\mathcal{H}$ spanned by vectors $|\mathbf{1}\rangle, |\mathbf{2}\rangle$, and $h_{jk}^{LOY} = \langle \mathbf{j}|H_{LOY}|\mathbf{k}\rangle$, $M_{jk}^{LOY} = \langle \mathbf{j}|M_{LOY}|\mathbf{k}\rangle$, $\Gamma_{jk}^{LOY} = \langle \mathbf{j}|\Gamma_{LOY}|\mathbf{k}\rangle$. Thus the time evolution in $\mathcal{H}_{||}$ is described by solutions of the Schrödinger–like equation

$$i\frac{\partial}{\partial t}|\psi; t\rangle_{\|} = H_{LOY}|\psi; t\rangle_{\|}, \quad (t \geq t_0), \tag{11}$$

where $|\psi; t\rangle_{\|} = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle$ belongs to the subspace $\mathcal{H}_{\|} \subset \mathcal{H}$.

The eigenvectors, $|K_S\rangle, |K_L\rangle$, for H_{LOY} to the eigenvalues, $\mu_S = m_S - \frac{i}{2}\gamma_S$ and $\mu_L = m_L - \frac{i}{2}\gamma_L$, have the following form

$$|K_S\rangle \; = \; \frac{1}{(|p_S|^2 + |q_S|^2)^{\frac{1}{2}}}\; (p_S\; |K_0\rangle \; - q_S\; |\overline{K}_0\rangle), \tag{12}$$

and

$$|K_L\rangle \; = \; \frac{1}{(|p_L|^2 + |q_L|^2)^{\frac{1}{2}}}\; (p_L\; |K_0\rangle \; + q_L\; |\overline{K}_0\rangle). \tag{13}$$

Now, if one assumes that the total system under considerations is CPT–invariant,

$$[\Theta, H] = 0, \tag{14}$$

where Θ is an antiunitary operator:

$$\Theta \stackrel{\text{def}}{=} \mathcal{CPT}, \tag{15}$$

and $\mathcal{C}$ is the charge conjugation operator, $\mathcal{P}$ — space inversion, and the antiunitary operator $\mathcal{T}$ represents the time reversal operation, one easily finds from (8) that in such a case the diagonal matrix elements of H_{LOY} must be equal:

$$h_{11}^{LOY} = h_{22}^{LOY}. \tag{16}$$

One of consequences of the property (16) is that in CPT invariant systems $p_S = p_L \equiv p$, $q_S = q_L \equiv q$ in (12), (13) and

$$\Big(\frac{q}{p} \Big)^2 \; = \; \frac{h_{21}^{LOY}}{h_{12}^{LOY}} \; = \; \text{const.} \tag{17}$$

Thus, if the CPT symmetry holds then

$$|K_S\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}} \; (p \; |K_0\rangle \; - q \; |\overline{K}_0\rangle), \tag{18}$$

and

$$|K_L\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}} \; (p \; |K_0\rangle \; + q \; |\overline{K}_0\rangle), \tag{19}$$

which causes that in this case

$$\langle K_S|K_L\rangle \equiv [\langle K_S|K_L\rangle]^* = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}. \tag{20}$$

Within this approach there is $|\frac{q}{p}| \neq 1$ in CPT invariant system when CP is violated [10]. This property and properties (16) – (20) are the standard result of the LOY approach and this is the picture which one meets in the literature [1] – [7]. The problem is that Khalfin shown that $\frac{q}{p} \neq$ const when CPT symmetry holds and CP does not [11] – [18].

Note that if one describes the properties of neutral mesons and the time evolution of their state vectors using the LOY method then, in fact, one assumes that the selfadjoint Hamiltonians $H, H^{(0)}$ and $H^{(1)}$ acting in $\mathcal{H}$ exist

and that the solutions of Schrödinger equation (1) describe the time evolution of states in $\mathcal{H}$. There is no LOY method and no LOY approximation without these Hamiltonians and without the Schrödinger equation.

This talk is based on the paper [19]. The aim of the talk is to confront the main predictions of the LOY theory such as (16), (17), (20), etc., with predictions following from the rigorous treatment of two state quantum mechanical subsystems and from the properties of the exact effective Hamiltonian for such subsystems. Sec. 2 contains the proof of the Khalfin's Theorem. In Sec. 3 properties of the the time evolution governed by a time independent effective Hamiltonian acting in two–dimensional subspace and of the evolution operator for this case are analyzed and confronted with the conclusions following from the Khalfin's Theorem. In Sec. 4 the properties of the exact effective Hamiltonian for two–state subsystems and consequences of the above mentioned Theorem are discussed. In Sec. 5 using a model of neutral kaon complex the results of calculations showing how the Khalfin's Theorem "works" are presented graphically. Section 6 contains final remarks.

2 Khalfin's Theorem

According to the general principles of quantum mechanics transitions of the system from a state $|\psi_1\rangle \in \mathcal{H}$ at time $t = 0$ to the state $|\psi_2\rangle \in \mathcal{H}$ at time $t > 0$, $|\psi_1\rangle \stackrel{t}{\rightarrow} |\psi_2\rangle$, are realized by the transition unitary unitary transition operator $U(t)$ acting in $\mathcal{H}$, such that

$$U(t_1)\,U(t_2) \;=\; U(t_1 + t_2) \;= U(t_2)\,U(t_1). \tag{21}$$

From this condition and from the unitarity it follows that

$$U(0) = \mathbb{I} \quad \text{and} \quad [U(t)]^{-1} \;\equiv\; [U(t)]^{+} \;=\; U(-t), \tag{22}$$

where $\mathbb{I}$ is the unit operator in $\mathcal{H}$.

The probability to find the system in the state $|\psi_j\rangle$ at time t if it was earlier at instant $t = 0$ in the initial state $|\psi_k\rangle$ is determined by the transition amplitude $A_{jk}(t)$,

$$A_{jk}(t) = \langle\psi_j|U(t)|\psi_k\rangle, \tag{23}$$

where $(j, k = 1, 2)$. Using (22) and following [14] it is easy to find that

$$[A_{12}(-t)]^{*} \;=\; A_{21}(t). \tag{24}$$

So, defining the function [14]

$$f_{21}(t) \stackrel{\text{def}}{=} \frac{A_{21}(t)}{A_{12}(t)}, \tag{25}$$

and taking into account the general property (24) one finds that the function $f_{21}(t)$ must satisfy the relation

$$[f_{21}(-t)]^* \, f_{21}(t) \; = \; 1. \tag{26}$$

Note that this last relation as well as the property (24) are valid for any two states $|\psi_1\rangle$, $|\psi_2\rangle \in \mathcal{H}$.

The Kalfin's Theorem concerns one of the basic properties of any two state subsystems and, in fact, it is not limited to only such subsystems as the neutral meson complexes. This Theorem states that [11] – [18]

Khalfin's Theorem
If

$$f_{21}(t) \; = \rho \; = \; \text{const.} \tag{27}$$

then there must be

$$R = |\rho| = 1. \tag{28}$$

Indeed, from (26) it follows that if $f_{21}(t) \; = \rho \; = \;$ const for every $t \geq 0$ then $[f_{21}(t')]^* \; = \; \zeta \; = \;$ const. for all $t' \leq 0$. Now, if the functions $f_{21}(t)$ and $[f_{21}(t')]^*$ are continuous at $t = t' = 0$ then there must be

$$R \; = |\rho| \; = \; |\zeta| \; = \; 1,$$

which is the proof of the Khalfin's Theorem.

The only problem in the above proof is to find conditions guaranteing the continuity of $f_{21}(t)$ at $t = 0$. There are two possibilities. The first: vectors $|\psi_1\rangle, |\psi_2\rangle$ are not orthogonal,

$$\langle \psi_1 | \psi_2 \rangle \neq 0. \tag{29}$$

and the second one: these vectors are orthogonal

$$\langle \psi_j | \psi_k \rangle = \delta_{jk}, \quad (j, k = 1, 2). \tag{30}$$

The case (29) is simple. One can always write

$$|\psi_2\rangle = |\psi_2\rangle_{||} + |\psi_2\rangle_{\perp}, \tag{31}$$

where

$$\langle\psi_1|\psi_2\rangle_{||} \neq 0, \quad \text{and} \quad \langle\psi_1|\psi_2\rangle_{\perp} = 0, \tag{32}$$

In such a case from (22), (23) it follows that $A_{21}(0) = {}_{||}\langle\psi_2|\psi_1\rangle = [\langle\psi_1|\psi_2\rangle_{||}]^* \neq 0$ and thus $A_{12}(0) \equiv [A_{21}(0)]^* \neq 0$ which yields

$$\lim_{t\to 0+} f_{21}(t) = \frac{[\langle\psi_1|\psi_2\rangle_{||}]^*}{\langle\psi_1|\psi_2\rangle_{||}} \stackrel{\text{def}}{=} \rho_1, \tag{33}$$

where $|\rho_1| = 1$, and

$$\lim_{t'\to 0-} [f_{21}(t')]^* \equiv \frac{1}{\rho_1}. \tag{34}$$

These last two relations mean that in the considered case (29) functions $f_{21}(t)|_{t\geq 0}$ as well as $[f_{21}(t')]^*|_{t'\leq 0}$ are continuous at $t = t' = 0$

Now let us concentrate the attention on the case (30). This situation occurs in the case of the neutral meson complexes but also it can be met in other cases. In general vectors $|\psi_1\rangle, |\psi_2\rangle$ need not describe the states of the neutral meson–antimeson pairs.

In the case presently considered (30), from (22), (23) and (30) one can see that $A_{21}(0) = 0$ and $A_{12}(0) = 0$ which by (25) means that without some additional conditions the function $f_{21}(t)$ need not be continuous at $t = 0$. Taking into account that quantum theory requires $U(t)$ to have the form,

$$U(t) = e^{-itH}, \tag{35}$$

(using units $\hbar = 1$), where H is the total hermitian Hamiltonian of the system, (or, in the interaction picture

$$U_I(t) = \mathbb{T}\, e^{-i\int_0^t H_I(\tau)\, d\tau}, \tag{36}$$

where $\mathbb{T}$ denotes the usual time ordering operator and $H_I(\tau)$ is the operator H in the interaction picture), one can easily verify that to assure the continuity of $f_{21}(t)$ at $t = 0$ it suffices that there exist such $n \geq 1$ that

$$\langle\psi_2|H^k|\psi_1\rangle = 0, \qquad (0 \leq k < n), \tag{37}$$

$$\langle\psi_2|H^n|\psi_1\rangle \neq 0 \quad \text{and} \quad |\langle\psi_2|H^n|\psi_1\rangle| < \infty. \tag{38}$$

Assuming that this property holds and using the d'Hospital rule one finds that simply

$$\lim_{t \to 0+} f_{21}(t) = \frac{\langle\psi_2|H^n|\psi_1\rangle}{\langle\psi_1|H^n|\psi_2\rangle}, \tag{39}$$

which means that $f_{21}(t)|_{t\geq 0}$ is continuous at $t = 0$. Similarly, the continuity of $[f_{21}(t')]^*|_{t'\leq 0}$ at $t' = 0$ is assured.

One of aims of this paper is to consider the consequences of the Khalfin's Theorem for neutral meson complexes. In the case of neutral mesons $\psi_1 = K_0,\, B_0,\, D_0 \ldots$ and $\psi_2 = \overline{K}_0,\, \overline{B}_0,\, \overline{D}_0 \ldots$. Thus in a general case the subspace of states of neutral mesons, $\mathcal{H}_{||}$, is a two–dimensional subspace of $\mathcal{H}$ spanned by orthogonal vectors $|\psi_1\rangle$, $|\psi_2\rangle$. For neutral meson complexes according to the experimental results the particle–antiparticle transitions $|\psi_1\rangle \rightleftharpoons |\psi_2\rangle$ exist, which means that there must exist $n < \infty$ such that the relation (38) occurs. (It is known form experiments that the transitions $|\Delta S| = 2$ exist, so in this case $n \leq 2$). This means that in fact for the neutral meson complexes, where the transitions $|\psi_1\rangle \rightleftharpoons |\psi_2\rangle$ take place, only the assumption of unitarity of the exact transition operator $U(t)$ assures the validity of the Khalfin's Theorem and no more assumptions (eg. of type that CPT symmetry holds in the total system under considerations) are required.

3 Properties of time evolution governed by a time–independent Hamiltonian acting in two state subspace

In this and subsequent Sections we will assume that the two–dimensional subspace $\mathcal{H}_\parallel$ of $\mathcal{H}$ is spanned by orthogonal vectors $|\psi_1\rangle$, $|\psi_2\rangle$, (30). So let us assume that the evolution operator $U_\parallel(t)$ acting in this $\mathcal{H}_\parallel$ has the following form

$$U_\parallel(t) = e^{-itH_\parallel}, \tag{40}$$

and that the operator $H_\parallel$ is a non–hermitian time–independent (2×2) matrix acting in $\mathcal{H}_\parallel$,

$$\frac{\partial h_{jk}}{\partial t} = 0, \tag{41}$$

where $h_{jk} = \langle\psi_j|H_\parallel|\psi_k\rangle$, $(j,k=1,2)$. It is obvious that the operator $U_\parallel(t)$ is the (2×2) matrix and

$$U_\parallel(t_1)\,U_\parallel(t_2) \;=\; U_\parallel(t_2)\,U_\parallel(t_1) \;=\; U_\parallel(t_1+t_2), \tag{42}$$

and

$$U_\parallel(0) \;=\; \mathbb{I}_\parallel,$$

where $\mathbb{I}_\parallel$ is the unit matrix in $\mathcal{H}_\parallel$.

It is easy to verify that the operator $U_\parallel(t)$ is the solution of the Schrödinger–like evolution equation for the subspace $\mathcal{H}_\parallel$,

$$i\frac{\partial}{\partial t}\,U_\parallel(t)\,|\psi\rangle_\parallel \;=\; H_\parallel\,U_\parallel(t)|\psi\rangle_\parallel, \qquad U_\parallel(0) \;=\; \mathbb{I}_\parallel, \tag{43}$$

where $|\psi\rangle_\parallel \in \mathcal{H}_\parallel$. Note that this last equation is the equation of the same type as the evolution equation used within the Lee–Oehme–Yang theory to describe the time evolution in neutral mesons subspace of states.

Using Pauli matrices σ_x, σ_y, σ_z the matrix $H_\parallel$ can be expressed as follows [20]

$$H_\parallel = h_0\,\mathbb{I}_\parallel + \vec{h}\,\cdot\,\vec{\sigma}, \tag{44}$$

where

$$\vec{h}\,\cdot\,\vec{\sigma} = h_x\,\sigma_x \,+\, h_y\,\sigma_y \,+\, h_z\,\sigma_z,$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and $h_0 = \frac{1}{2}(h_{11}+h_{22})$. Within the use of the relation (44) the operator $U_\parallel(t)$ given by (40) can be rewritten in the following form

$$\begin{aligned} U_\parallel(t) \;&=\; e^{-itH_\parallel} \equiv u_0(t)\,\mathbb{I}_\parallel \,+\, \vec{u(t)}\,\cdot\,\vec{\sigma} \\ &\equiv\; e^{-ith_0}\,[\mathbb{I}_\parallel\,\cos{(th)} \,-\, i\,\frac{\vec{h}\,\cdot\,\vec{\sigma}}{h}\,\sin{(th)}], \end{aligned} \tag{45}$$

where

$$\begin{aligned} u_0(t) \;&=\; \frac{1}{2}(u_{11}(t)+u_{22}(t)), \\ u_{jk} \;&\stackrel{\text{def}}{=}\; \langle\psi_j|U_\parallel(t)|\psi_k\rangle, \quad (j,k=1,2), \\ \vec{u(t)}\,\cdot\,\vec{\sigma} \;&=\; u_x(t)\,\sigma_x \,+\, u_y(t)\,\sigma_y \,+\, u_z(t)\,\sigma_z, \\ h^2 \;&=\; \vec{h}\,\cdot\,\vec{h} \;=\; h_x^2 \,+\, h_y^2 \,+\, h_z^2. \end{aligned} \tag{46}$$

Now taking into account that simply (see (44)),

$$\vec{h} \cdot \vec{\sigma} \equiv H_{\|} - h_0 \, \mathbb{I}_{\|}, \tag{47}$$

from (45) one finds

$$\begin{aligned} u_{12}(t) &= -i \, e^{-ith_0} \, \frac{h_{12}}{h} \, \sin(th), && (48) \\ u_{21}(t) &= -i \, e^{-ith_0} \, \frac{h_{21}}{h} \, \sin(th), && (49) \\ u_{11}(t) &= e^{-ith_0} \, [\cos(th) - i \, \frac{h_z}{h} \, \sin(th)], && (50) \\ u_{22}(t) &= e^{-ith_0} \, [\cos(th) + i \, \frac{h_z}{h} \, \sin(th)], && (51) \end{aligned}$$

where, $h_z = \frac{1}{2}(h_{11} - h_{22})$.

Relations (48) and (49) yield

$$\frac{u_{21}(t)}{u_{12}(t)} \equiv \frac{h_{21}}{h_{12}} \stackrel{\text{def}}{=} r = \text{const}. \tag{52}$$

Another useful relation following from (50) and (51) is the following one

$$u_{11}(t) - u_{22}(t) = -2i \, e^{-ith_0} \, \frac{h_z}{h} \, \sin(th). \tag{53}$$

So if one has any time–independent effective Hamiltonian $H_{\|}$ acting in $\mathcal{H}_{\|}$ and the evolution operator $U_{\|}(t)$ for $\mathcal{H}_{\|}$ has the form $U_{\|}(t) = e^{-itH_{\|}}$ then

$$u_{11}(t) = u_{22}(t) \; \Leftrightarrow \; h_{11} = h_{22}. \tag{54}$$

This property is quite independent of relations of type (52).

All the above properties, including (52), (54), are true for every time–independent effective Hamiltonian $H_{||}$ acting in two–dimensional subspace $\mathcal{H}_{||}$. In other words, they hold for the LOY effective Hamiltonian, H_{LOY}, as well as for every $H_{||} \neq H_{LOY}$.

The conclusion following from Khalfin's Theorem, (27), (28) and from (52) seems to be important,

Conclusion 1
If $|r| \neq 1$ and the time–independent effective Hamiltonian $H_{||}$ is the exact effective Hamiltonian for the subspace $\mathcal{H}_{||}$ of states of neutral mesons, so that

$$u_{jk}(t) \equiv A_{jk}(t), \tag{55}$$

where $j \neq k$, $(j, k = 1, 2)$, r is defined by (52) and $u_{jk}(t)$, $A_{jk}(t)$ are given by (46) and (23) respectively, then the evolution operator $U(t)$ for the total state space $\mathcal{H}$ can not be a unitary one.

Indeed, experimental results indicate that for the neutral kaon complex $|r| \neq 1$ (see, e.g. [10]). So, this conclusion holds because from the Khalfin's Theorem it follows that if $|r| \neq 1$ and matrix elements $A_{jk}(t)$, $(j, k = 1, 2)$ are the matrix elements of the exact evolution operator $U(t)$ then there must be $|r| \neq$ const. Thus if the relation (55) is the true relation then there is only one possibility: The Khalfin's Theorem is not valid in this case. From the proof of this Theorem given in the previous Section and analysis of the case of neutral mesons performed there it follows that this Theorem holds if the evolution operator $U(t)$ for the total state space $\mathcal{H}$ of the system containing two state subsystem under considerations is a unitary operator. For the neutral mesons subsystem Khalfin's Theorem need not hold only if the total evolution operator $U(t)$ is not a unitary operator.

4 Symmetries CP, CPT and the exact evolution operator and effective Hamiltonian for neutral mesons subsystem

The exact (transition) evolution operator for the subspace $\mathcal{H}_{||}$ can be found using the projection operator, P, defining this subspace, $\mathcal{H}_{||} = P\mathcal{H}$. Projector P can be constructed by means of orthonormal vectors $|\psi_1\rangle$, $|\psi_2\rangle$,

$$P = |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|. \tag{56}$$

The exact transition operator for $\mathcal{H}_{||}$ is given by the nonzero (2×2) submatrix, $\mathbf{A}(t)$, of the operator $PU(t)P$, where $U(t)$ is the exact transition operator

(35) for the total state space $\mathcal{H}$ of the system containing neutral mesons subsystem. So,

$$\mathbf{A}(t) = \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix}, \tag{57}$$

where $A_{jk}(t) = \langle \psi_j | U(t) | \psi_k \rangle$, $(j, k = 1, 2)$, and $\mathbf{A}(0) = \mathbb{I}_{\|}$. Note that the matrix $\mathbf{A}(t)$ is not unitary. Within the use of this exact transition operator for the subspace $\mathcal{H}_{\|}$ the exact effective Hamiltonian $H_{\|}$ governing the time evolution in $\mathcal{H}_{\|}$ can be expressed as follows [21, 22, 23, 24, 25, 26]

$$H_{\|} \,=\, H_{\|}(t) \,\equiv\, i \frac{\partial \mathbf{A}(t)}{\partial t} [\mathbf{A}(t)]^{-1}. \tag{58}$$

Thus the exact evolution equation for the subspace $\mathcal{H}_{\|}$ has the Schrödinger–like form (43), (11), with time–dependent effective Hamiltonian (58),

$$i \frac{\partial}{\partial t} |\psi, t\rangle_{\|} \,=\, H_{\|}(t)\, |\psi, t\rangle_{\|}, \tag{59}$$

where, $|\psi, t\rangle_{\|} \,=\, a_1(t)\, |\psi_1\rangle \,+\, a_2(t)\, |\psi_2\rangle \,=\, \mathbf{A}(t)\, |\psi\rangle_{\|} \,\in\, \mathcal{H}_{\|}$ and $|\psi\rangle_{\|} = a_1 |\psi_1 + a_2 |\psi_2\rangle \in \mathcal{H}_{\|}$ is the initial state of the system, $\| \, |\psi\rangle_{\|} \, \| = 1$.

It is easy to find from (58) the general formulae for the diagonal matrix elements, h_{jj}, as well as for the off–diagonal matrix elements, h_{jk} of the exact $H_{\|}(t)$. We have [25]

$$\begin{aligned} h_{11}(t) \;&=\; \frac{i}{\det \mathbf{A}(t)} \Big(\frac{\partial A_{11}(t)}{\partial t} A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \Big), && (60) \\ h_{22}(t) \;&=\; \frac{i}{\det \mathbf{A}(t)} \Big(- \frac{\partial A_{21}(t)}{\partial t} A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t} A_{11}(t) \Big), && (61) \end{aligned}$$

and so on. Using (60), (61) the difference $(h_{11} - h_{22}) = 2 h_z$ playing an important role in relations (53), (54) can be expressed as follows [25]

$$\begin{aligned} h_{11}(t) - h_{22}(t) \;&=\; i \frac{1}{\det \mathbf{A}(t)} \Big\{ A_{11}(t)\, A_{22}(t)\, \frac{\partial}{\partial t} \ln \Big(\frac{A_{11}(t)}{A_{22}(t)} \Big) \\ &\quad + A_{12}(t)\, A_{21}(t)\, \frac{\partial}{\partial t} \ln \Big(\frac{A_{21}(t)}{A_{12}(t)} \Big) \Big\}. \end{aligned} \tag{62}$$

Now let us analyze some consequences of the conservation or violation of CP–, CPT–symmetries in the total system under considerations. If we

assume that the system is CPT invariant, that is that (14) holds, then one easily finds that for neutral meson complex, (that is for $|\psi_1\rangle \equiv |\mathbf{1}\rangle, |\psi_2\rangle \equiv |\mathbf{2}\rangle$), [11, 12, 13, 16, 25, 27]

$$A_{11}(t) = A_{22}(t). \tag{63}$$

The assumption (14) gives no relations between $A_{12}(t)$ and $A_{21}(t)$.

If the system under considerations is assumed to be CP invariant,

$$[\mathcal{CP}, H] = 0, \tag{64}$$

then using the following, most general, phase convention

$$\mathcal{CP}|\mathbf{1}\rangle = e^{-i\alpha}|\mathbf{2}\rangle, \quad \mathcal{CP}|\mathbf{2}\rangle = e^{+i\alpha}|\mathbf{1}\rangle, \tag{65}$$

(instead of the standard one: $\mathcal{CP}|\mathbf{1}\rangle = -|\mathbf{2}\rangle$, $\mathcal{CP}|\mathbf{2}\rangle = -|\mathbf{1}\rangle$) one easily finds that for the diagonal matrix elements of the matrix $\mathbf{A(t)}$ the relation (63) holds in this case also, and that there are,

$$A_{12}(t) = e^{2i\alpha} A_{21}(t), \tag{66}$$

for the off–diagonal matrix elements and

$$A_{11}(t) = A_{22}(t), \tag{67}$$

for diagonal matrix elements.

This means that if the CP symmetry is conserved in the system containing the subsystem of neutral mesons, then for every $t > 0$ there must be

$$|\,\frac{A_{21}(t)}{A_{12}(t)}\,| = 1 \;\equiv\; \text{const.} \tag{68}$$

On the other hand, when CP symmetry is violated,

$$[\mathcal{CP}, H] \neq 0, \tag{69}$$

then one can prove that in a such system the modulus of the ratio $\frac{A_{21}(t)}{A_{12}(t)}$ must be different from 1 for every $t > 0$,

$$[\mathcal{CP}, H] \neq 0 \quad \Rightarrow \quad |\,\frac{A_{21}(t)}{A_{12}(t)}| \neq 1, \quad (\forall t > 0). \tag{70}$$

The proof of this property is rigorous (see [26]).

Let us examine the consequences of the assumption that CPT invariance of the total system under considerations has the same consequences for the properties of the matrix elements of the exact effective Hamiltonian for neutral meson subsystem and for the matrix elements of H_{LOY}. Strictly speaking, let us analyze the implications of the assumptions that if the CPT symmetry holds then the property (16) occurs in real system, i.e. that the diagonal matrix elements of the effective Hamiltonian are equal. It means that we should verify under which conditions the property $(h_{11}(t) - h_{22}(t)) = 0$ is admissible for the exact effective Hamiltonian for $t > 0$. So, starting from the expression (62), then using relations (63), (68), (70) and the Khalfin's Theorem (28) the following conclusions can be drawn [26]:

Conclusion 2

If $(h_{11}(t) - h_{22}(t)) = 0$ for $t > 0$ then there must be

a)

$$\frac{A_{11}(t)}{A_{22}(t)} = \text{const.}, \quad \text{and} \quad \frac{A_{21}(t)}{A_{12}(t)} = \text{const.}, \quad (\text{for } \ t > 0),$$

or,

b)

$$\frac{A_{11}(t)}{A_{22}(t)} \neq \text{const.}, \quad \text{and} \quad \frac{A_{21}(t)}{A_{12}(t)} \neq \text{const.}, \quad (\text{for } \ t > 0).$$

The following interpretation of a) and b) follows from (63), (68), (70) and from the Khalfin's Theorem (28). Case a) means that CP–symmetry is conserved and there is no information about CPT invariance. Case b) denotes that the system under considerations is neither CP–invariant nor CPT–invariant.

In our discussion the CPT Theorem [28] — [31] can not be neglected. The CPT Theorem is a fundamental theorem of axiomatic quantum field theory. It follows from locality, Lorentz invariance and unitarity. One should also take into account another fact that there is no an experimental evidence that CPT symmetry is violated [10]. Therefore, the assumption that any quantum theory of elementary particles should be CPT invariant seems to be obvious. So, let us assume that CPT symmetry is the exact symmetry of the system under considerations, that is that the condition (14) holds. In such a case the relation (63) holds. The consequence of this is that the

expression (62) becomes simpler and it is easy to prove that the following property must hold [25]

$$h_{11}(t) - h_{22}(t) = 0 \;\;\Leftrightarrow\;\; \frac{A_{21}(t)}{A_{12}(t)} \;=\; \text{const.}, \;\; (t > 0). \tag{71}$$

Taking into account the Khalfin's Theorem, (28), and relations (63), (70) one finds that the following property must hold in the case of the exact effective Hamiltonian for neutral meson subsystem:

Conclusion 3
If $[\Theta, H] = 0$ and $[\mathcal{CP}, H] \neq 0$, that is if $A_{11}(t) = A_{22}(t)$ and $\left|\frac{A_{21}(t)}{A_{12}(t)}\right| \neq 1$ for $t > 0$, then there must be $(h_{11}(t) - h_{22}(t)) \neq 0$ for $t > 0$.

So within the exact theory one can say that for real systems, the property (54) can not occur if CPT symmetry holds and CP is violated. This means that the relation (54) can only be considered as an approximation. The question is if such an approximation is sufficiently accurate in order to reflect real properties of neutral meson complexes. One potential solution to this problem is suggested in the next Section, where model calculations are discussed.

5 Model calculations

In this Section we will discuss results of numerical calculations performed within the use of the program "Mathematica" for the model considered by Khalfin in [11, 12] , and by Nowakowski in [16] and then used in [32, 33]. This model is formulated using the spectral language for the description of K_S, K_L and K^0, $\overline{K}^0$, by introducing a hermitian Hamiltonian, H, with a continuous spectrum of decay products labeled by α, β, etc.,

$$H|\phi_\alpha(m)\rangle = m\,|\phi_\alpha(m)\rangle, \quad \langle\phi_\beta(m')|\phi_\alpha(m)\rangle = \delta_{\alpha\beta}\delta(m' - m). \tag{72}$$

Here H is the mentioned total Hamiltonian for the system mentioned in Sections 1, 2 and 4. H includes all interactions and has absolutely continuous spectrum. We have

$$|K_S\rangle = \int_{\text{Spec }(H)} dm \, \sum_\alpha c_{S,\alpha}(m)|\phi_\alpha(m)\rangle, \tag{73}$$

$$|K_L\rangle = \int_{\text{Spec }(H)} dm \, \sum_{\beta} c_{S,\alpha}(m)|\phi_\beta(m)\rangle, \tag{74}$$

and

$$|\mathbf{j}\rangle = \int_{\text{Spec }(H)} dm \, \sum_{\alpha} c_{j,\alpha}(m)|\phi_\alpha(m)\rangle, \tag{75}$$

where $j = 1, 2$. Thus, the exact $A_{jk}(t)$ can be written as the Fourier transform of the density $\omega_{jk}(m)$, $(j, k = 1, 2)$,

$$A_{jk}(t) = \int_{-\infty}^{+\infty} dm \, e^{-imt} \omega_{jk}(m), \tag{76}$$

where

$$\omega_{jk}(m) = \sum_{\alpha} c^*_{j,\alpha}(m) \, c_{k,\alpha}(m). \tag{77}$$

The minimal mathematical requirement for $\omega_{jk}(m)$ is the following: $\int_{-\infty}^{+\infty} dm \, |\omega_{jk}(m)| < \infty$. Other requirements for $\omega_{jk}(m)$ are determined by basic physical properties of the system. The main property is that the energy (i.e. the spectrum of H) should be bounded from below, Spec$(H) = [m_g, \infty)$ and $m_g > -\infty$.

Starting from densities $\omega_{jk}(m)$ one can calculate $A_{jk}(t)$. In order to find these densities from relation (77) one should know the expansion coefficients $c_{j,\alpha}(m)$. Using physical states $|K_S\rangle, |K_L\rangle$ and relations (12), (13) they can be expressed in terms of the expansion coefficients $c_{S,\alpha}(m), c_{S,\alpha}(m)$. Thus, assuming the form of coefficients $c_{S,\alpha}(m), c_{S,\alpha}(m)$ defining physical states of neutral kaons one can compute all $A_{jk}(t)$, $(j, k = 1, 2)$.

The model considered by Khalfin is based on the assumption that (see formula (35) in [12]).

$$c_{S,\beta}(m) = \sqrt{\frac{\gamma_S}{2\pi}} \; \frac{\xi_{S,\beta}(m)}{|\xi_{S,\beta}(m_S - i\frac{\gamma_S}{2})|} \, \frac{a_{S,\beta}(K_S \rightarrow \beta)}{m - m_S + i\frac{\gamma_S}{2}}, \tag{78}$$

$$c_{L,\beta}(m) = \sqrt{\frac{\gamma_L}{2\pi}} \, \frac{\xi_{L,\beta}(m)}{|\xi_{S,\beta}(m_L - i\frac{\gamma_L}{2})|} \, \frac{a_{L,\beta}(K_L \rightarrow \beta)}{m - m_L + i\frac{\gamma_L}{2}}, \tag{79}$$

where $a_{S,\beta}$ and $a_{L,\beta}$ are the decay (transition) amplitudes and $\xi_{S(L),\beta}(m)$ are, in general, some nonsingular "preparation functions".

The calculation performed in [16] uses Khalfin's assumption made for simplicity in [12] that $\xi_{S(L),\beta}(m) = 1$, strictly speaking, an assumption is used that there is

$$\frac{\xi_{S(L),\beta}(m)}{|\xi_{S(L),\beta}(m_{S(L)} - i\frac{\gamma_{S(L)}}{2})|} \equiv \Theta(m - m_g) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } m \geq m_g, \\ 0 & \text{if } m < m_g, \end{cases}, \tag{80}$$

in (78), (79). Within this assumption one obtains, for example, that

$$\mathcal{A}_{SS}(t) \stackrel{\text{def}}{=} \langle K_S|e^{-itH}|K_S\rangle = \int_{-\infty}^{+\infty} dm\, \omega_{SS}(m)\, e^{-itm}, \tag{81}$$

where

$$\omega_{SS}(m) = \Theta(m - m_g)\, \frac{\gamma_S}{(m - m_S)^2 + \frac{\gamma_S^2}{4}}\, \frac{S}{2\pi}, \tag{82}$$

$$S = \sum_{\alpha} |a_{S,\alpha}(K_S \rightarrow \alpha)|^2, \tag{83}$$

and so on.

For simplicity, it is assumed in [16] that $m_g = 0$. So all integrals of type (81) and (76) are taken between the limits $m = 0$ and $m = +\infty$. In [16] all these assumptions made it possible to find analytically amplitudes of type $A_{jk}(t)$ and to express them in terms of known special functions such as integral exponential functions and related. The same assumptions were used in [32] (see [32], relations (37) – (39) and (42) – (47)) and will be used in this paper. Note that putting $\Theta(m - m_g) \equiv 1$ in (82) leads to a strictly exponential form of amplitudes of type $\mathcal{A}_{SS}(t)$ as functions of time t. On the other hand, keeping $\Theta(m)$ in the assumed simplest physically admissible form (80) results in the presence of additional nonoscillatory terms in amplitudes of type $\mathcal{A}_{SS}(t), \mathcal{A}_{LL}(t)$ etc. and thus in amplitudes $A_{jk}(t)$ as well (see [16, 32, 33]).

The results obtained within this model and presented below are obtained assuming that CPT symmetry holds (i.e. that relations (63) are valid in the model considered) but CP symmetry is violated and by inserting into (79) — (82) and related formulae the following values of the parameters characterizing neutral kaon complex: $m_S \simeq m_L \simeq m_{average} = 497.648 MeV$, $\Delta m = 3.489 \times 10^{-12} MeV$, $\tau_S = 0.8935 \times 10^{-10} s$, $\tau_L = 5.17 \times 10^{-8} s$, $\gamma_L =$

$1.3 \times 10^{-14} MeV$, $\gamma_S = 7.4 \times 10^{-12} MeV$ [10]. This model together with the above data make it possible to examine numerically the Khalfin's Theorem as well as other relations and conclusions obtained using this Theorem (for details see [16, 32, 33]).

The results of numerical calculations of the modulus of the ratio $\frac{A_{12}(t)}{A_{21}(t)}$ for some time interval are presented below in Fig. 1.

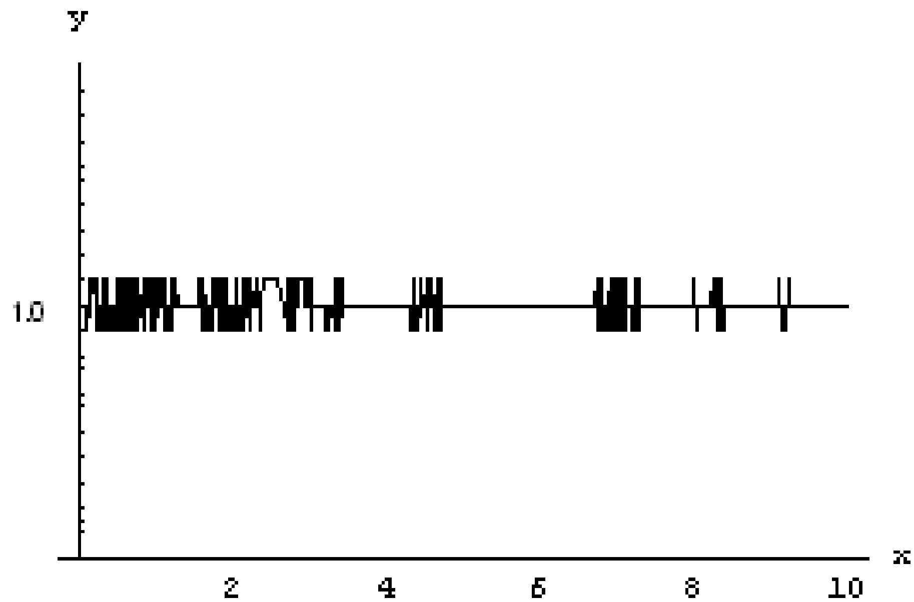

Figure 1: Numerical examination of the Khalfin's Theorem.

Here $y(x) = |r(t)| \equiv |\frac{A_{21}(t)}{A_{12}(t)}|$, $x = \frac{\gamma_L}{\hbar} \cdot t$, and $x \in (0.01, 10)$.

Analyzing the results of the calculations presented graphically in Fig. 1 one can find that for $x \in (0.01, 10)$,

$$y_{max}(x) - y_{min}(x) \simeq 3.3 \times 10^{-16}, \tag{84}$$

where

$$\begin{aligned} y_{max}(x) &= |r(t)|_{max}, \\ y_{min}(x) &= |r(t)|_{min}. \end{aligned} \tag{85}$$

So from Fig. 1 and (85) the conclusion follows that if one is able to measure the modulus of the ratio $\frac{A_{12}(t)}{A_{21}(t)}$ only up to the accuracy 10^{-15} then one

sees this quantity as a constant function of time. The variations in time of $\mid \frac{A_{12}(t)}{A_{21}(t)} \mid$ become detectable for the experimenter only if the accuracy of his measurements is of order 10^{-16} or better.

Similarly, using "Mathematica" and starting from the amplitudes $A_{jk}(t)$ and using the relation (62) and the condition (63) one can compute the difference $(h_{11}(t) - h_{22}(t))$ for the model considered. Results of such calculations for some time interval are presented below in Fig. 2, 3. An expansion of scale in Fig. 2 shows that continuous fluctuations, similar to those in Fig. 3, appear.

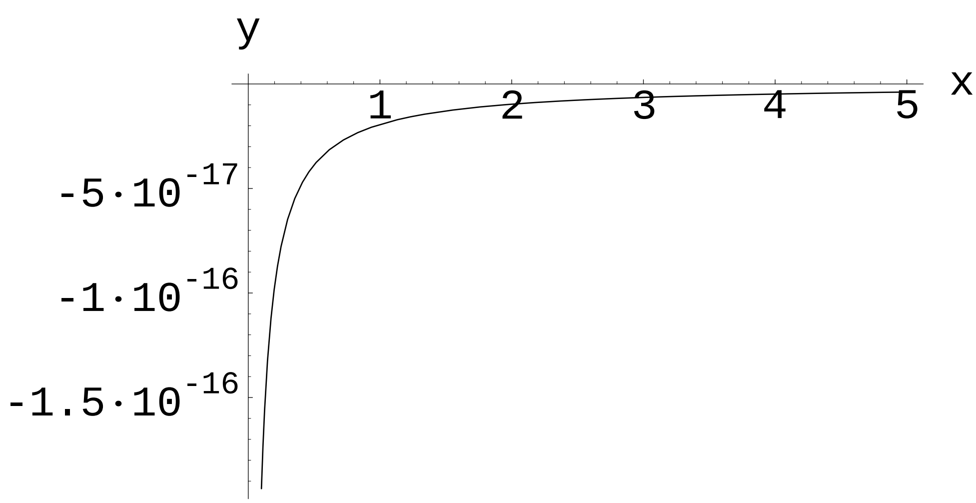

Figure 2: Real part of $(h_{11}(t) - h_{22}(t))$

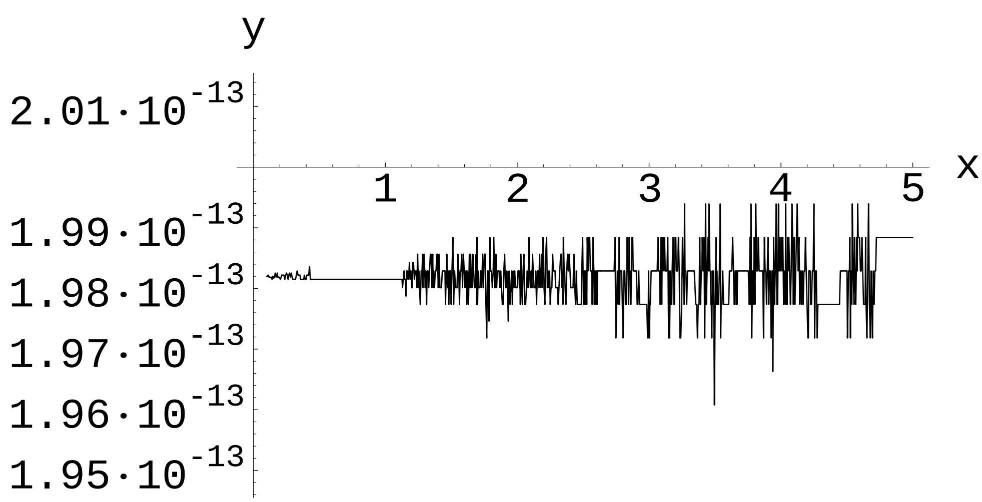

Figure 3: Imaginary part of $(h_{11}(t) - h_{22}(t))$

There is $y(x) = \Re\,(h_{11}(t) - h_{22}(t)$ and $y(x) = \Im\,(h_{11}(t) - h_{22}(t)$ in Figs 2, 3 respectively. In these Figures $x = \frac{\gamma_L}{\hbar} \cdot t$, $x \in (0.01, 5.0)$ and $\Re\,(z)$ and $\Im\,(z)$ denote the real and imaginary parts of z respectively and units on the y–axis are in [MeV].

One can compare the results presented in Figs. 2, 3 with the result obtained analytically. Within the model considered the analytical formulae for the matrix elements $h_{jk}(t)$, $(j, k = 1, 2)$, were obtained in [32]. Inserting the experimental values of τ_L, μ_L, μ_S, etc., mentioned above it is found in [32] for $t = \tau_L$ that

$$\Re\,(h_{11}(t \sim \tau_L) - h_{22}(t \sim \tau_L)) \simeq -4.771 \times 10^{-18} MeV, \tag{86}$$

$$\Im\,(h_{11}(t \sim \tau_L) - h_{22}(t \sim \tau_L)) \simeq 7.283 \times 10^{-16} MeV \tag{87}$$

and

$$\frac{|\Re\,(h_{11}(t \sim \tau_L) - h_{22}(t \sim \tau_L))|}{m_{average}} \equiv \frac{m_{K^0} - m_{\bar{K^0}}}{m_{average}} \sim 10^{-21}, \tag{88}$$

There is a visible difference between the results presented in Figs. 2, 3 and in (86) — (88). It may be attributed to finite accuracy of numerical

calculations performed by Mathematica. No approximations have been used in the analytical calculations.

6 Final remarks

Let us analyze consequences of the results contained in Sec. 2 - 5 for the standard picture of CP violation or possible CPT violation effects in the neutral meson complex. The attention will be focused on the neutral kaon complex as the best studied subsystem of neutral mesons. The form of parameters usually used to describe the scale of CP– and CPT–violation effects depends on the phase used in relations (65) defining the action of $\mathcal{CP}$ operator on the states of neutral K mesons. So, in order to define these parameters it is convenient to choose a phase convention for this operator. For simplicity the following phase convention for neutral kaons is commonly used

$$\mathcal{CP}|\mathbf{1}\rangle = (-1)|\mathbf{2}\rangle, \qquad \mathcal{CP}|\mathbf{2}\rangle = (-1)|\mathbf{1}\rangle, \tag{89}$$

instead of the general one (65). Within this phase convention one finds that vectors

$$|K_{1(2)}\rangle \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}(\,|\mathbf{1}\rangle - (+)|\mathbf{2}\rangle), \tag{90}$$

are normalized, orthogonal

$$\langle K_j|K_k\rangle = \delta_{jk}, \quad (j,k = 1,2), \tag{91}$$

eigenvectors of $\mathcal{CP}$ transformation (89),

$$\mathcal{CP}|K_{1(2)}\rangle = +(-1)|K_{1(2)}\rangle, \tag{92}$$

for the eigenvalues $+1$ and -1 respectively.

Using these eigenvectors $|K_{1(2)}\rangle$ of the CP–transformation vectors $|K_L\rangle$ and $|K_S\rangle$ can be expressed as follows [3, 34, 35]

$$|K_{L(S)}\rangle \equiv \frac{1}{\sqrt{1+|\varepsilon_{l(s)}|^2}}\Big(\,|K_{2(1)}\rangle + \varepsilon_{l(s)}|K_{1(2)}\rangle\Big), \tag{93}$$

where

$$\varepsilon_l \;=\; \frac{h_{12}-h_{11}+\mu_L}{h_{12}+h_{11}-\mu_L} \equiv -\frac{h_{21}-h_{22}+\mu_L}{h_{21}+h_{22}-\mu_L}, \tag{94}$$

$$\varepsilon_s \;=\; \frac{h_{12}+h_{11}-\mu_S}{h_{12}-h_{11}+\mu_S} \equiv -\frac{h_{21}+h_{22}-\mu_S}{h_{21}-h_{22}+\mu_S}, \tag{95}$$

The form (93) of $|K_L\rangle$ and $|K_S\rangle$ is used in many papers in which possible departures from CP– or CPT–symmetry in the system considered are discussed. Within the standard approach the following parameters are used to describe the scale of CP– and possible CPT – violation effects [3, 34, 35]:

$$\varepsilon \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s + \varepsilon_l) \equiv \frac{h_{12} - h_{21}}{D}, \tag{96}$$

$$\delta \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s - \varepsilon_l) \equiv \frac{h_{11} - h_{22}}{D} \equiv \frac{2h_z}{D}, \tag{97}$$

where

$$D \stackrel{\text{def}}{=} h_{12} + h_{21} + \Delta\mu, \tag{98}$$

and $\Delta\mu = \mu_S - \mu_L$. According to the standard interpretation following from the LOY approximation, ε describes violations of CP–symmetry and δ is considered as a CPT–violating parameter [3, 34, 35]. Such an interpretation of these parameters follows from the properties of LOY theory of time evolution in the subspace of neutral kaons [2] — [6], [27], [34, 35].

The relation (93) leads to the following formula for the product $\langle K_S|K_L\rangle$,

$$\langle K_S|K_L\rangle \equiv N(\varepsilon_s^* + \varepsilon_l), \tag{99}$$

where $N = N^* = [(1 + |\varepsilon_s|^2)(1 + |\varepsilon_l|^2)]^{-1/2}$. By means of the parameters δ and ε the product (99) can be expressed as follows

$$\langle K_S|K_L\rangle \equiv 2N(\Re\,\varepsilon - i\,\Im\,\delta). \tag{100}$$

There is

$$\delta \simeq \frac{h_{11} - h_{22}}{2(\mu_s - \mu_l)} \equiv \delta_{||}\, e^{i\phi_{SW}} + \delta_{\perp}\, e^{i(\phi_{SW} + \pi/2)}, \tag{101}$$

in the case of $|\varepsilon_s| \ll 1$ and $|\varepsilon_l| \ll 1$ (see, eg. [10], pp. 623 – 644). Here ϕ_{SW} is the superweak phase, $\tan\,\phi_{SW} = \frac{2(m_l - ms)}{\gamma_s - \gamma_l}$, and

$$\delta_{||} = \frac{1}{4}\frac{\Gamma_{11} - \Gamma_{22}}{\sqrt{(m_s - m_l)^2 + \frac{1}{4}(\gamma_s - \gamma_l)^2}}, \tag{102}$$

$$\delta_{\perp} = \frac{1}{2}\frac{\Re\,(h_{11} - h_{22})}{\sqrt{(m_s - m_l)^2 + \frac{1}{4}(\gamma_s - \gamma_l)^2}}, \tag{103}$$

are the real parameters. Thus

$$\Im\,\delta = \delta_{||}\,\sin\,\phi_{SW}\;+\;\delta_{\perp}\,\cos\,\phi_{SW}. \tag{104}$$

The consequence of (16) is that in CPT invariant but CP noninvariant system $\delta_{||} = \delta_{||}^{LOY} = 0$ and $\delta_{\perp} = \delta_{\perp}^{LOY} = 0$ which leads to the standard result $\Im\delta^{LOY} = 0$ (here δ^{LOY} denotes the parameter δ, (101), calculated for $H_{||} = H_{LOY}$). From this property and (100) the conclusion that the product $\langle K_S|K_L\rangle$ must be real is drawn in the literature. This conclusion is considered as the standard result. Note that in the light of the main result of Sec. 4, *Conclusion 3* and from the results of the model calculations presented in Sec. 5 (see Fig 2 and Fig 3), such a conclusion seems to be wrong in the case of the exact effective Hamiltonian $H_{||}$, that is, in the case of the exact theory. From *Conclusion 3* and Figs 2, 3 one infers that there must be $\delta_{\perp} \neq 0$, and $\delta_{||} \neq 0$ in the case of CPT invariant but CP noninvariant system and therefore there must be $\Im\,\delta \neq 0$ (see (104)) in such a system. This means that the right hand side of the relation (100) is a complex number and therefore in the case of conserved CPT– and violated CP–symetries, in contrast with the standard result, there must be $\langle K_S|K_L\rangle \neq \langle K_S|K_L\rangle^*$ in the real systems.

Note that the property $\langle K_S|K_L\rangle = \langle K_S|K_L\rangle^*$ play an important role when one applies the Bell–Steinberger unitarity relations [36] for designing or interpreting tests with neutral mesons. So in the light of the above discussion results obtained in such a way should not be considered as a conclusive evidence, especially when subtle effects, such as the possible CPT violations, are studied.

From the *Conclusion 3* from Sec. 4 and from the results of the model calculations presented in Sec. 5 it also follows that the parameter δ should not be considered as the parameter measuring the scale of possible CPT violation effects: In the more accurate approach [37] and in the exact theory one obtains $\delta \neq 0$ for every system with violated CP symmetry and this property occurs quite independently of whether this system is CPT invariant or not. What is more, from the *Conclusion 3* one finds that if CP symmetry is violated and CPT symmetry holds then there must be $\varepsilon_l \neq \varepsilon_s$ (see (97)) contrary to the standard predictions of the LOY theory. These conclusions are in full agreement with the results obtained in [38] within the quantum field theory analysis of binary systems such as the neutral meson complexes.

It seems that the results following from the Khalfin's Theorem and discussed in Sec. 3 – 5 have a particular meaning for such attempts to test

Quantum Mechanics and CPT invariance in the neutral kaon complex as those described in [39, 40] and recently in [41]. Simply the expected magnitude of the possible effects analyzed in these papers is very close to the results presented in Sec. 5 and obtained within the more accurate treatment of the neutral kaon subsystem. Generally, in the light of the results discussed in Sec. 2 - 5, the interpretation of tests of such tiny effects as the possible CPT violation effects and a similar one based on the LOY approximation should not be considered as conclusive.

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